



NICHOLAS SCHOOL OF THE  
ENVIRONMENT AND EARTH SCIENCES  
DUKE UNIVERSITY



# **ENVIRON 761:** Fuzzy Logic and GIS

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# What is fuzzy logic?



Who in this picture is **tall**?

Who is **short**?

Who is **average**?

# What is fuzzy logic?

- Binary logic → Output is 1 (true) or 0 (false)

Tall > 6'

Name	Height	Tall (binary)
Natalie Portman	5'3"	0
Scarlett Johansson	5'3"	0
Haley Atwell	5'7"	0
Mark Ruffalo	5'8"	0
Cobie Smulders	5'8"	0
Robert Downy Jr.	5'8.5"	0
Don Cheadle	5'8.5"	0
Jeremy Renner	5'10"	0
Chris Evans	6'	0
Tom Hiddleston	6'2"	1
Chris Pratt	6'2"	1
Samuel L. Jackson	6'2.5"	1
Chris Hemsworth	6'2.75"	1
Dave Bautista	6'4.5"	1

# What is fuzzy logic?

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- Fuzzy logic is an alternative to binary logic.
- Outputs are not limited to the sets of **1** and **0**.
  - e.g. **Tall** or **not tall**
- Instead, it assigns *fuzzy* values based on degree of membership to the sets of 1 and 0...
  - e.g. **80% tall** & **20% not tall**
- Fuzzy values are assigned based on fuzzy membership functions...

# What is fuzzy logic?

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*Sorites paradox:*

If I remove one sand grain from a pile, it's still a pile,  
but if I carry on, it's soon not a pile...



*At what point is a pile no longer a pile?*

# What is fuzzy logic?

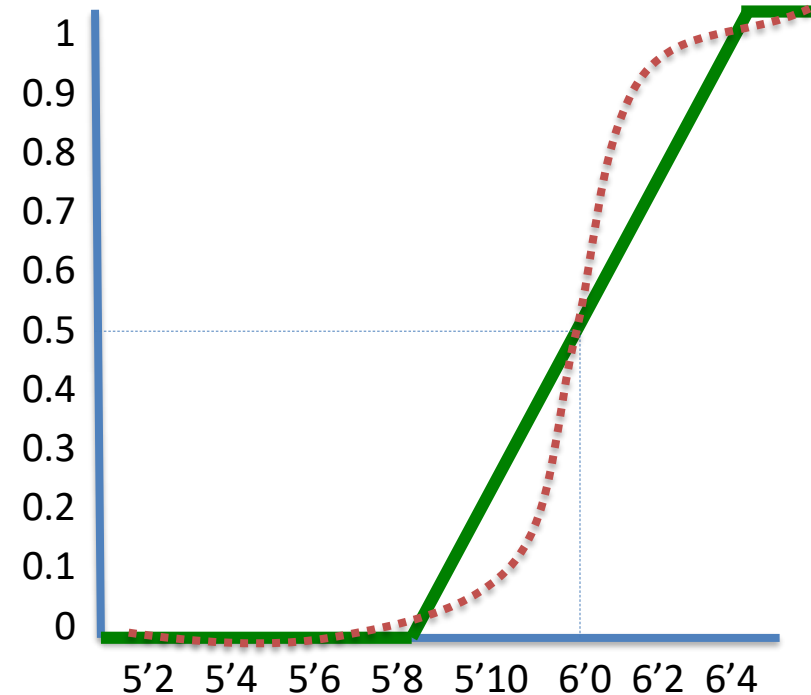
Tall > 6'

Name	Height	Tall (binary)	Tall (fuzzy)	
Natalie Portman	5'3"	0	0.00	} Definitely not tall
Scarlett Johansson	5'3"	0	0.00	
Haley Atwell	5'7"	0	0.00	
Mark Ruffalo	5'8"	0	0.20	
Cobie Smulders	5'8"	0	0.20	
Robert Downy Jr.	5'8.5"	0	0.25	
Don Cheadle	5'8.5"	0	0.25	
Jeremy Renner	5'10"	0	0.30	
Chris Evans	6'	0	0.50	
Tom Hiddleston	6'2"	1	0.60	} Mostly tall
Chris Pratt	6'2"	1	0.60	
Samuel L. Jackson	6'2.5"	1	0.70	
Chris Hemsworth	6'2.75"	1	0.80	
Dave Bautista	6'4.5"	1	1.00	→ Definitely tall

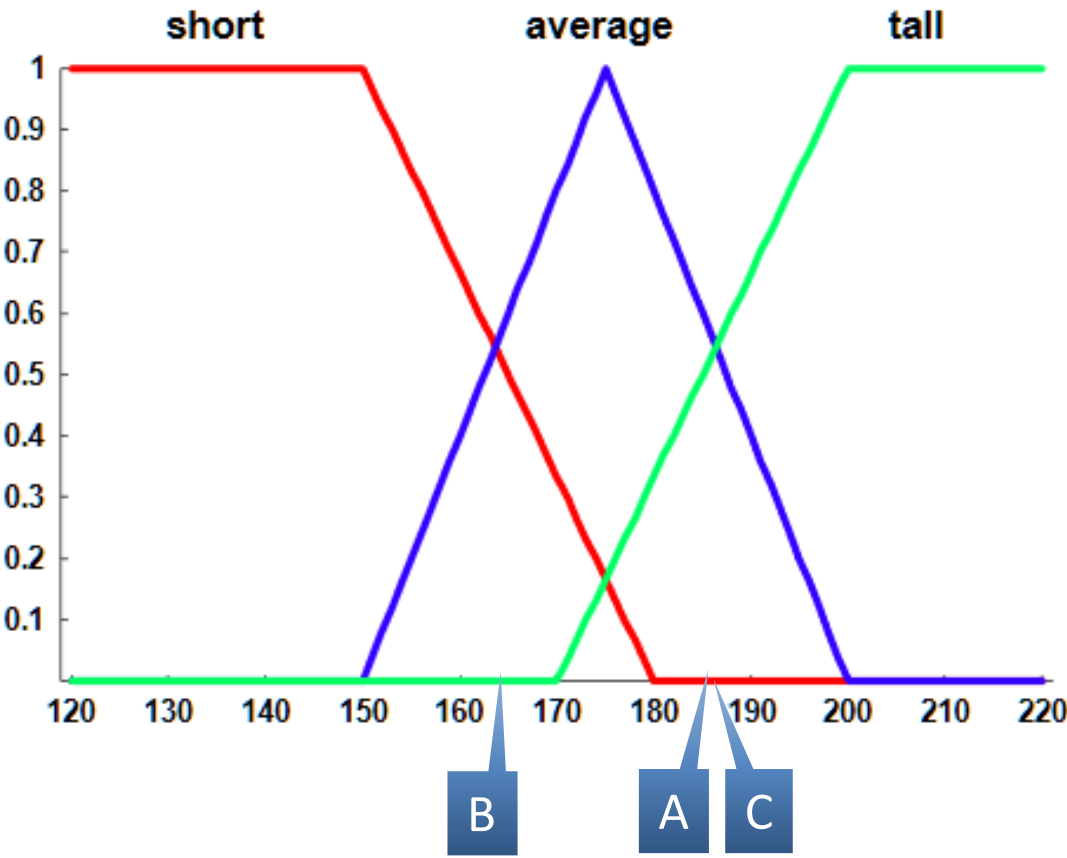
# How are fuzzy values calculated?

- Answer: *Membership functions*

Name	Height	Tall (binary)	Tall (fuzzy)
Natalie Portman	5'3"	0	0.00
Scarlett Johansson	5'3"	0	0.00
Haley Atwell	5'7"	0	0.00
Mark Ruffalo	5'8"	0	0.20
Cobie Smulders	5'8"	0	0.20
Robert Downy Jr.	5'8.5"	0	0.25
Don Cheadle	5'8.5"	0	0.25
Jeremy Renner	5'10"	0	0.30
Chris Evans	6'	0	0.50
Tom Hiddleston	6'2"	1	0.60
Chris Pratt	6'2"	1	0.60
Samuel L. Jackson	6'2.5"	1	0.70
Chris Hemsworth	6'2.75"	1	0.80
Dave Bautista	6'4.5"	1	1.00



# Membership functions: multiple classes



Short: < 165cm  
 Average: > 165cm & < 185cm  
 Tall: > 185cm

A – 185cm  
 B – 165cm  
 C – 186cm

binary

	Short	Average	Tall
A	0	1	0
B	1	0	0
C	0	0	1

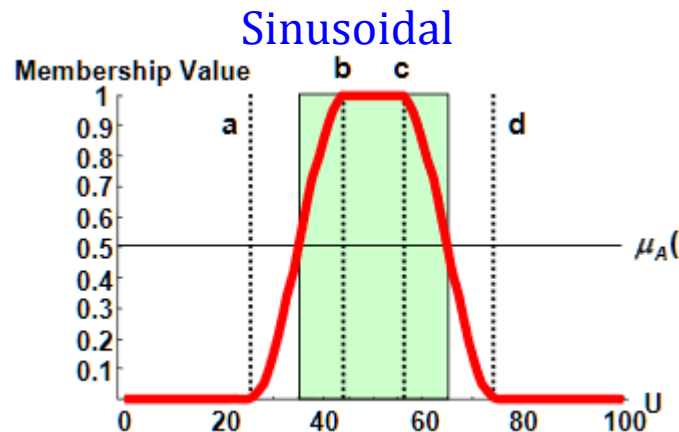
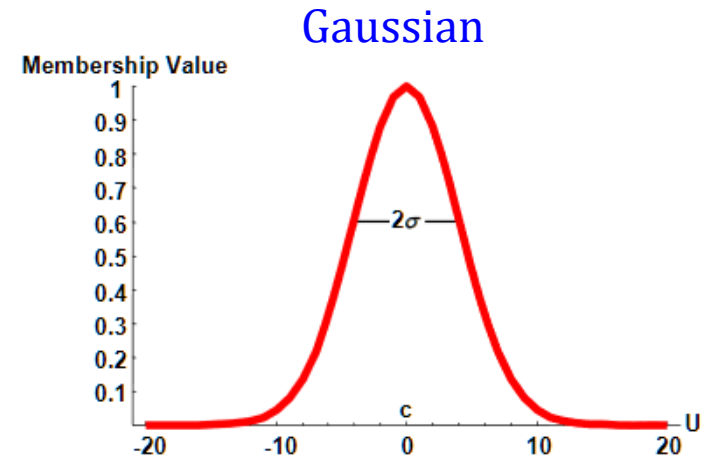
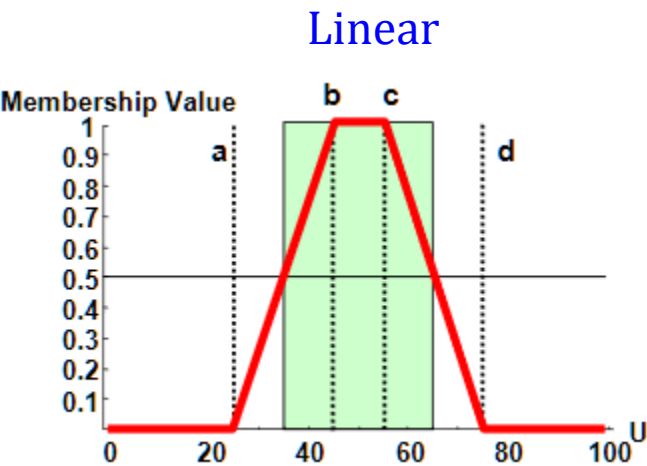
fuzzy

	Short	Average	Tall
A	0.00	0.60	0.50
B	0.50	0.60	0.00
C	0.00	0.56	0.53



# Fuzzy membership functions

Transform “crisp” values into values between 0 and 1, indicating strength of membership in a set...



# Fuzzy inference

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- In binary logic:
  - Premise 1: *If  $x = A$ , then  $y = B$*
  - Premise 2:  $x$  is  $A$
  - Conclusion:  $y$  is  $B$
  
- If grass is wet, it has rained.
- The grass is wet, therefore it has rained.



# Fuzzy inference

- In fuzzy logic:
  - Premise 1: *If  $x = A$ , then  $y = B$*
  - Premise 2:  *$x$  is  $A'$  (an acceptably likely member of  $A$ )*
  - Conclusion:  *$y$  is  $B'$  (an acceptably likely member of  $B$ )*
- If grass is long, I should mow.
- The grass is *fairly* long, therefore *maybe* I'll mow
- The grass is *really* long, I'd better mow right now!



# Fuzzy inference: more conditions

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*If grass is long and it isn't too hot, I should mow?*

Rules:

- Grass is long, not too hot: I should mow now!
- Grass is not very long, not too hot: I'll mow later.
- Grass is long, too hot: I may mow now.
- Grass is not very long, too hot: I'll mow later.

# Another example...

Risk analysis based on degrees of risk ranging from 1 (low risk) to 4 (very high risk).

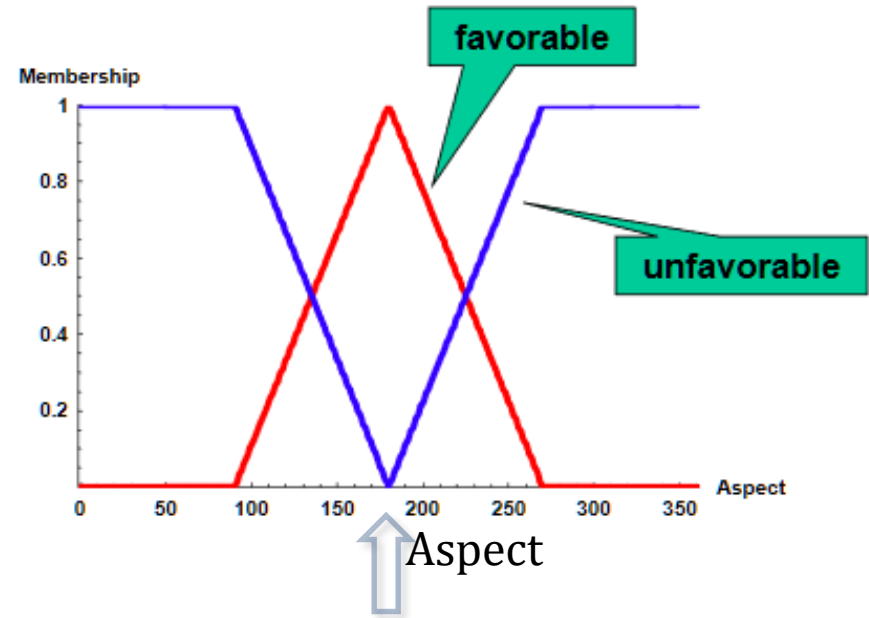
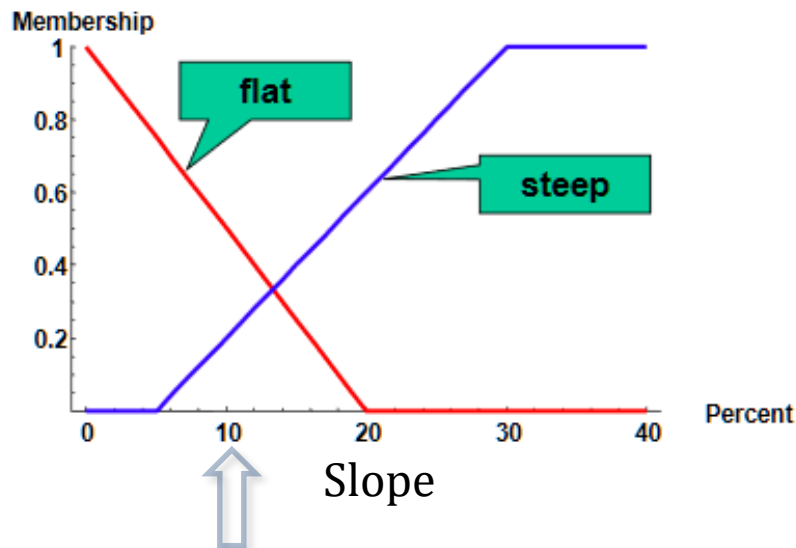
- If **slope is flat** & **aspect is favorable**, then risk is **1**.
- If **slope is steep** & **aspect is favorable**, then risk is **2**.
- If **slope is flat** & **aspect is unfavorable**, then risk is **1**.
- If **slope is steep** & **aspect is unfavorable**, then risk is **4**.

Fuzzy factors

Consequence

# Another example...

Fuzzy memberships:



Slope of 10% and aspect of 180°

Flat and Favorable  
 Steep and Favorable  
 Flat and Unfavorable  
 Steep and Unfavorable

	Slope (s)	Aspect (a)
Rule1	0.5	1
Rule2	0.2	1
Rule3	0.5	0
Rule4	0.2	0

# Computing overall scores..

## Consequences

Rule 1...risk is 1.  
 Rule 2...risk is 2.  
 Rule 3...risk is 1.  
 Rule 4...risk is 4.

- Compute the *minimum score* (conservative)
- “Conclusion” is (*score* \* *consequence*)

For a slope of 10 percent and an aspect of 180 degrees we have the following results:

	Slope (s)	Aspect (a)	Min(s,a)	Score	Conclusions
Rule1	0.5	1	0.5	1	0.5
Rule2	0.2	1	0.2	2	0.4
Rule3	0.5	0	0	1	0
Rule4	0.2	0	0	4	0

For the final result we get  $c' = \frac{0.5+0.4+0+0}{0.5+0.2+0+0} = 1.29$ , which means a low risk.

- “Defuzzification”: divide  $\Sigma(\text{conclusion})$  by  $\Sigma(\text{min score})$ 
  - It’s a weighted sum of the risk (1 thru 4 in this case)





# Bald eagle habitat criteria

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- *Land cover*
  - “Not *too sparse* and not *too dense*...”
  - Areas within 40 and 70% forest cover...
- *Water*
  - “Like to be *close to water*”
  - 500, 1000, 1500 m buffer...
- *Human disturbance*
  - “*Far from* from urban areas, roads, etc...”

# Bald eagle habitat criteria

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## Binary logic:

Resulting pixel values are either 1 or 0

Habitat = 1 if:

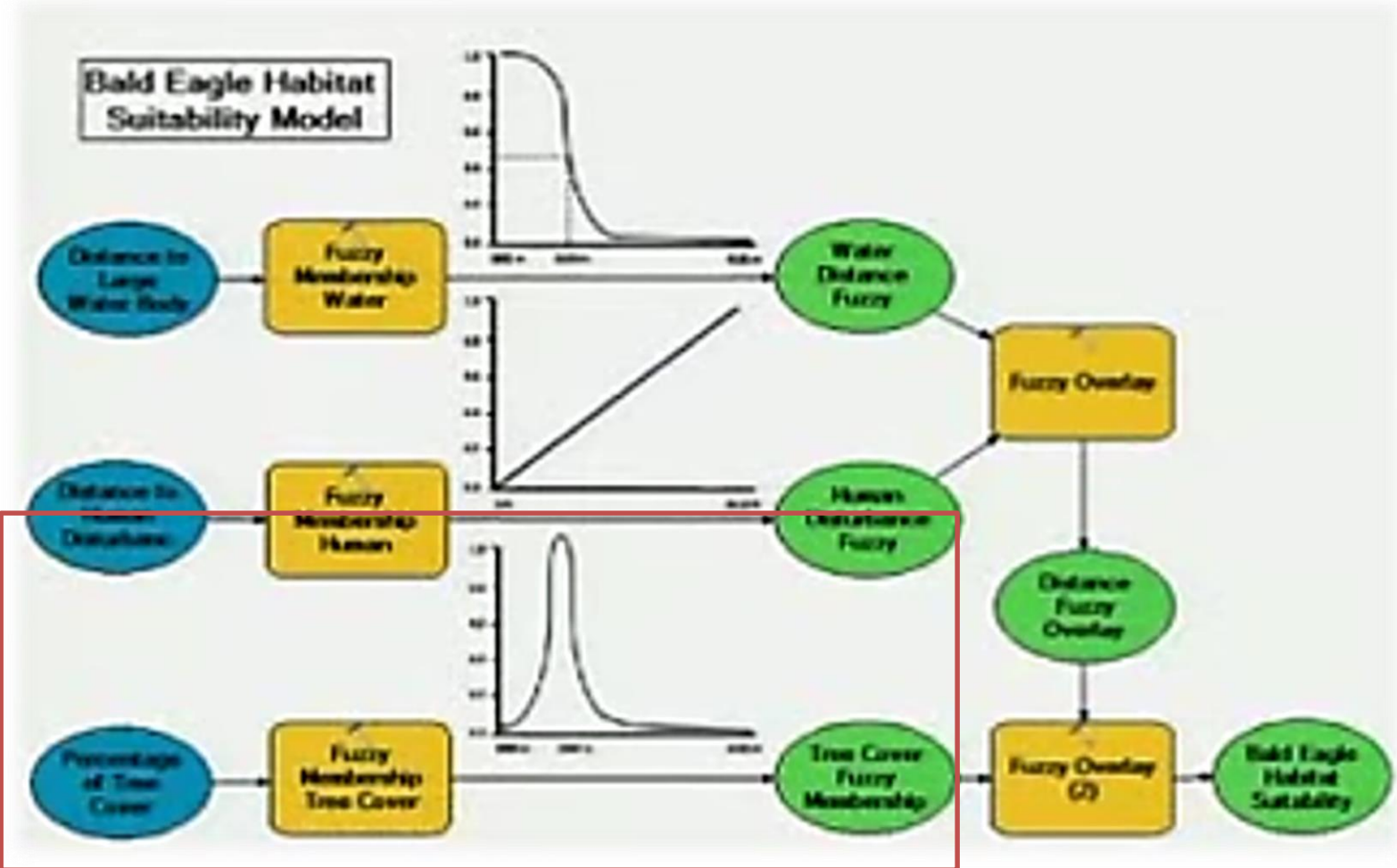
- > 40% forest  
&
- < 70% forest  
&
- < 1000 m of lake or river  
&
- > 1500 m from developed area

Otherwise, Habitat = 0

# Bald eagle habitat criteria

Fuzzy logic:

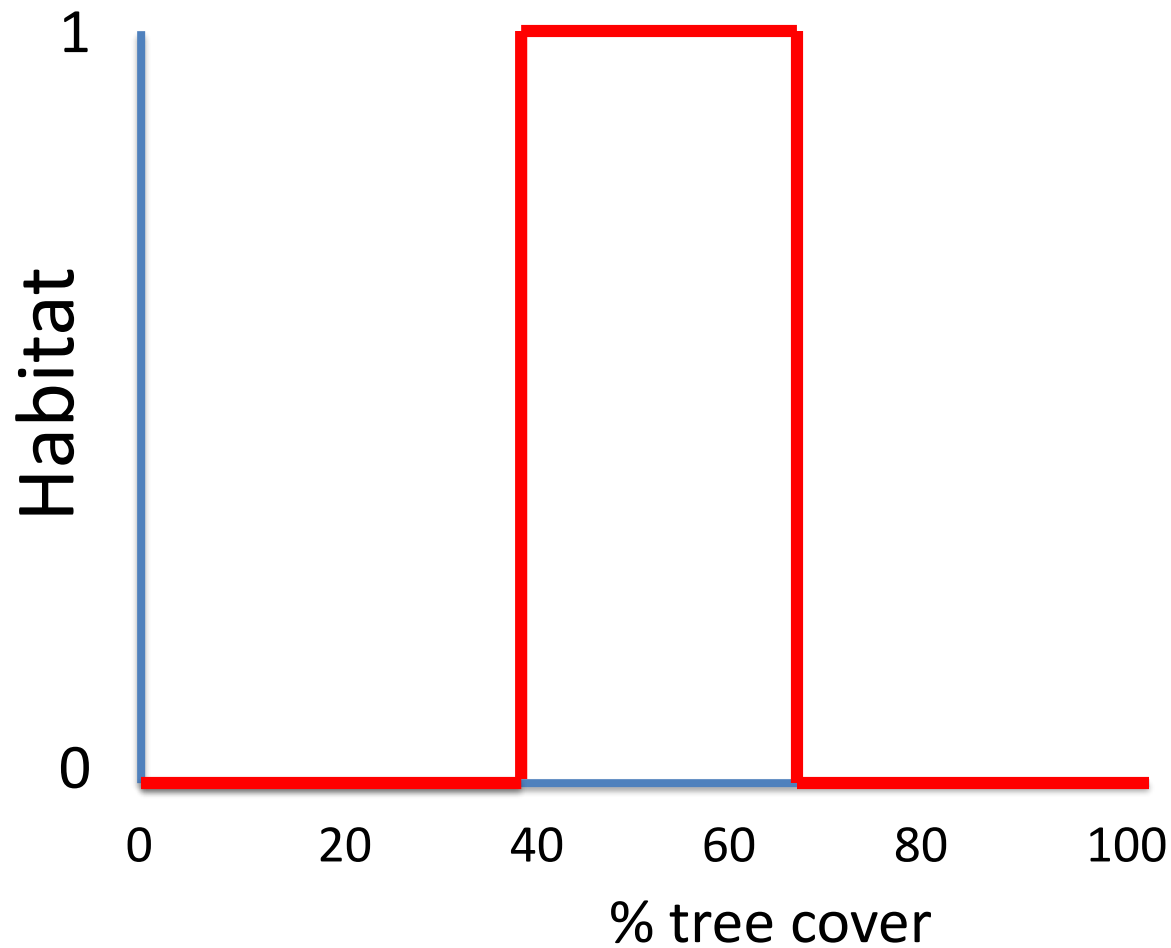
Results are a *continuum* between 1 and 0



# Bald eagle habitat criteria

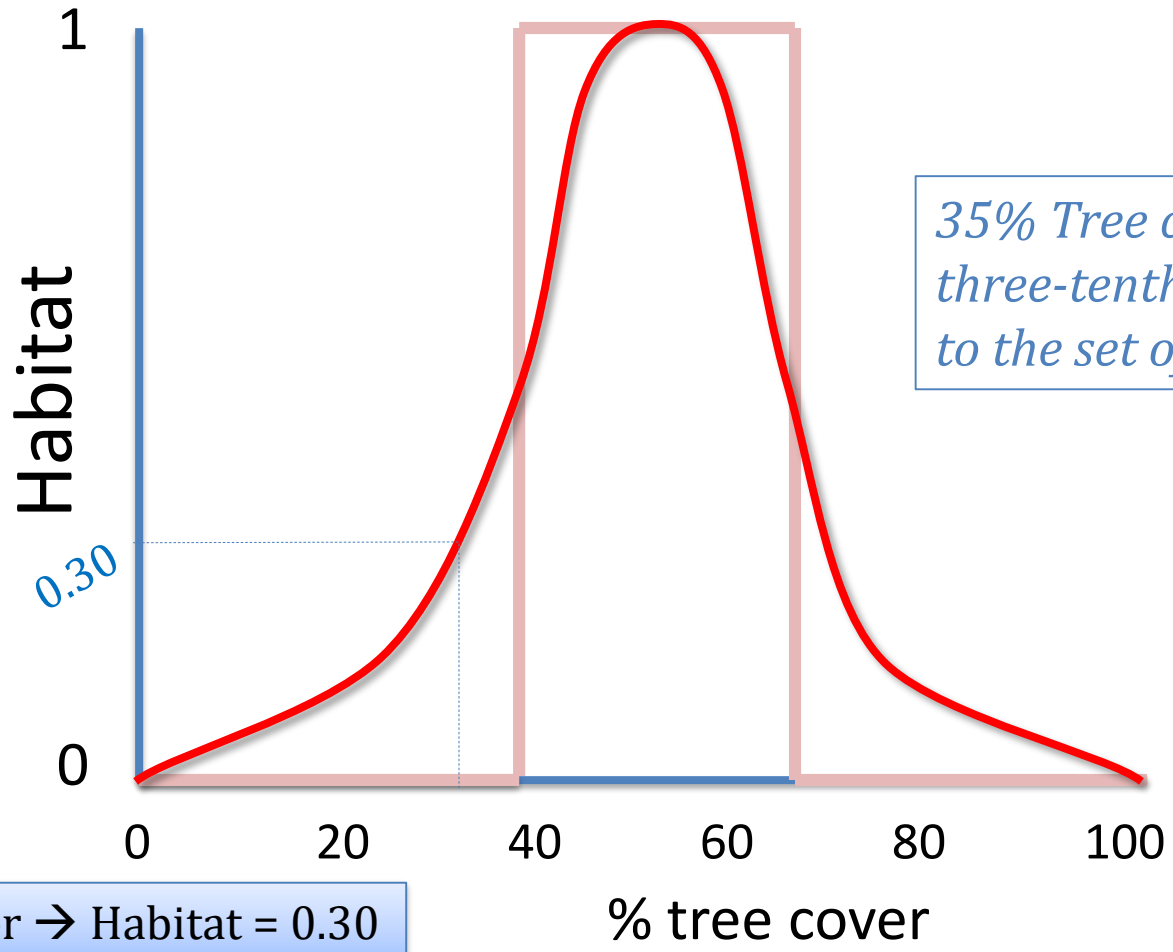
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## Binary logic - Forest:



# Bald eagle habitat criteria

## Fuzzy logic - Forest:



# Fuzzy membership functions - ArcMap

- [-] Spatial Analyst Tools
  - [+] Conditional
  - [+] Density
  - [+] Distance
  - [+] Extraction
  - [+] Generalization
  - [+] Groundwater
  - [+] Hydrology
  - [+] Interpolation
  - [+] Local
  - [+] Map Algebra
  - [+] Math
  - [+] Multivariate
  - [+] Neighborhood
  - [-] Overlay
    - Fuzzy Membership**
    - Fuzzy Overlay
    - Weighted Overlay
    - Weighted Sum

**Fuzzy Membership**

Input raster  
Euclidean Distance to Developed

Output raster  
V:\Lab5\_ThreatMapping\_Fuzzy\Scratch\FuzDist

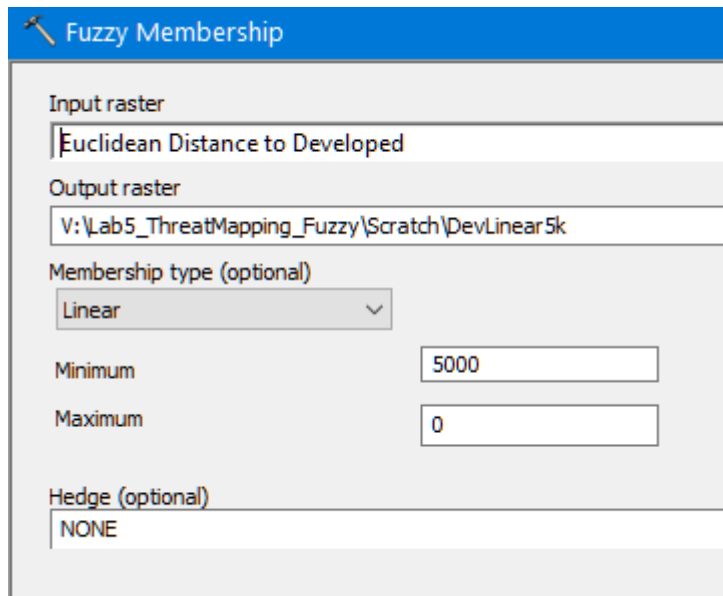
Membership type (optional)

MSSmall	
Gaussian	
Small	1
<b>Large</b>	1
Near	
MSSmall	
MSLarge	
Linear	
NONE	

<http://www.esri.com/news/arcuser/0410/fuzzy-logic-table2.pdf>

# Fuzzy membership functions - ArcMap

- Linear:
  - A linear increasing or decreasing membership between two inputs.



The screenshot shows the 'Fuzzy Membership' tool interface. The title bar is blue with a hammer icon and the text 'Fuzzy Membership'. The interface is divided into several sections:

- Input raster:** A text box containing 'Euclidean Distance to Developed'.
- Output raster:** A text box containing 'V:\Lab5\_ThreatMapping\_Fuzzy\Scratch\DevLinear5k'.
- Membership type (optional):** A dropdown menu with 'Linear' selected.
- Minimum:** A text box containing '5000'.
- Maximum:** A text box containing '0'.
- Hedge (optional):** A text box containing 'NONE'.

# Fuzzy analysis – Pronghorn

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- Criteria for unthreatened habitat:
  - Far from developed areas
  - Far from power lines
  - Few nearby roads
  - Mostly open land



# Fuzzy analysis – Pronghorn

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- Criteria
  - **Far** from developed areas
    - “Far”: > 5000 m, with linear response
  - **Far** from power lines
    - “Far”: > 6000 m, with sigmoidal response
  - **Few** nearby roads
    - “Few”: < 3km per, sq. km with linear response
  - **Mostly** open land cover
    - 60% to 90% herbaceous or scrub within 1km

# Conclusions

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- Fuzzy analysis allows us to soften the impact of somewhat arbitrary cutoffs (e.g. 1km from roads).
- We can do a bit of fuzzy analysis (and did do this) by reclassifying continuous values (e.g. distance from roads) into non-binary classes.
- Alternatively, we can apply fuzzy membership functions, but what function to use involves a different set of assumptions.
- We still need to interpret how to combine the outputs (fuzzy overlay).
- In short, fuzzy analysis doesn't really solve our problem of involving subjective analysis; the subjectivity is just used elsewhere.
- However, fuzzy analysis offers new ways to break down these problems and use decision analysis techniques in our methods.