



NICHOLAS SCHOOL OF THE
ENVIRONMENT AND EARTH SCIENCES
DUKE UNIVERSITY



ENVIRON 761: Fuzzy Logic and GIS

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What is fuzzy logic?



Who in this picture is **tall**?

Who is **short**?

Who is **average**?

What is fuzzy logic?

- Binary logic → Output is 1 (true) or 0 (false)

Tall > 6'

Name	Height	Tall (binary)
Natalie Portman	5'3"	0
Scarlett Johansson	5'3"	0
Haley Atwell	5'7"	0
Mark Ruffalo	5'8"	0
Cobie Smulders	5'8"	0
Robert Downy Jr.	5'8.5"	0
Don Cheadle	5'8.5"	0
Jeremy Renner	5'10"	0
Chris Evans	6'	0
Tom Hiddleston	6'2"	1
Chris Pratt	6'2"	1
Samuel L. Jackson	6'2.5"	1
Chris Hemsworth	6'2.75"	1
Dave Bautista	6'4.5"	1

What is fuzzy logic?

- Fuzzy logic is an alternative to binary logic.
- Outputs are not limited to the sets of **1** and **0**.
 - e.g. **Tall** or **not tall**
- Instead, it assigns *fuzzy* values based on degree of membership to the sets of 1 and 0...
 - e.g. **80% tall** & **20% not tall**
- Fuzzy values are assigned based on fuzzy membership functions...

What is fuzzy logic?

Sorites paradox:

If I remove one sand grain from a pile, it's still a pile,
but if I carry on, it's soon not a pile...



At what point is a pile no longer a pile?

What is fuzzy logic?

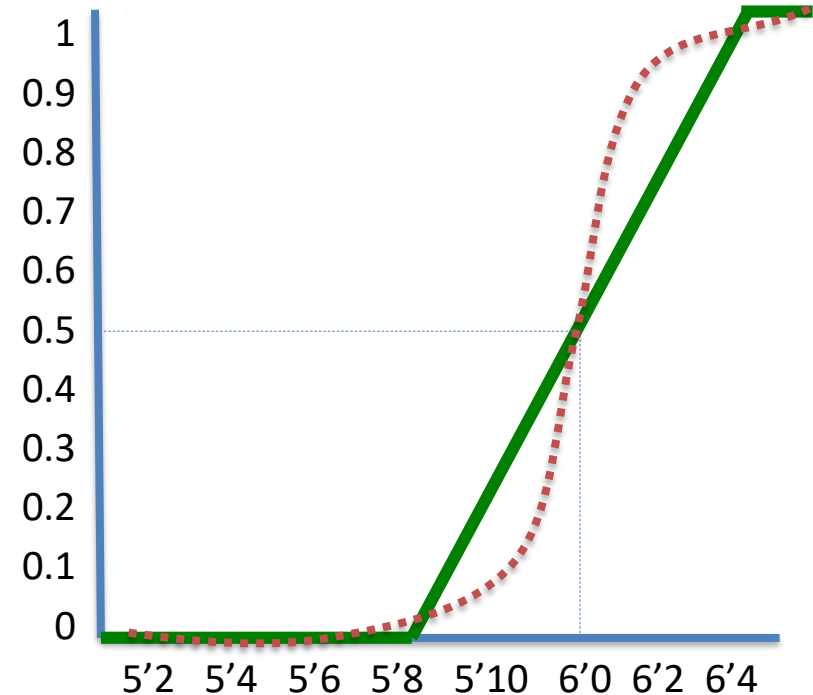
Tall > 6'

Name	Height	Tall (binary)	Tall (fuzzy)	
Natalie Portman	5'3"	0	0.00	} Definitely not tall
Scarlett Johansson	5'3"	0	0.00	
Haley Atwell	5'7"	0	0.00	
Mark Ruffalo	5'8"	0	0.20	
Cobie Smulders	5'8"	0	0.20	
Robert Downy Jr.	5'8.5"	0	0.25	
Don Cheadle	5'8.5"	0	0.25	
Jeremy Renner	5'10"	0	0.30	
Chris Evans	6'	0	0.50	
Tom Hiddleston	6'2"	1	0.60	} Mostly tall
Chris Pratt	6'2"	1	0.60	
Samuel L. Jackson	6'2.5"	1	0.70	
Chris Hemsworth	6'2.75"	1	0.80	
Dave Bautista	6'4.5"	1	1.00	→ Definitely tall

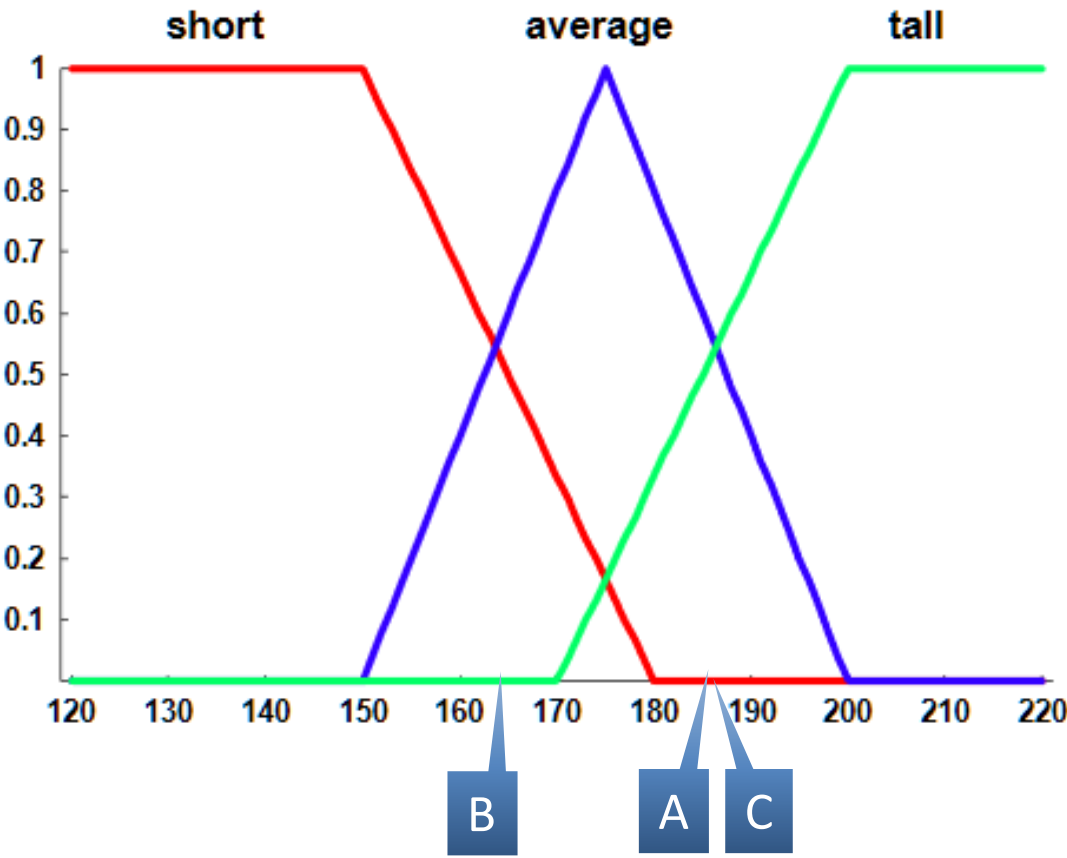
How are fuzzy values calculated?

- Answer: *Membership functions*

Name	Height	Tall (binary)	Tall (fuzzy)
Natalie Portman	5'3"	0	0.00
Scarlett Johansson	5'3"	0	0.00
Haley Atwell	5'7"	0	0.00
Mark Ruffalo	5'8"	0	0.20
Cobie Smulders	5'8"	0	0.20
Robert Downy Jr.	5'8.5"	0	0.25
Don Cheadle	5'8.5"	0	0.25
Jeremy Renner	5'10"	0	0.30
Chris Evans	6'	0	0.50
Tom Hiddleston	6'2"	1	0.60
Chris Pratt	6'2"	1	0.60
Samuel L. Jackson	6'2.5"	1	0.70
Chris Hemsworth	6'2.75"	1	0.80
Dave Bautista	6'4.5"	1	1.00



Membership functions: multiple classes



Short: < 165cm
 Average: > 165cm & < 185cm
 Tall: > 185cm

A – 185cm
 B – 165cm
 C – 186cm

binary

	Short	Average	Tall
A	0	1	0
B	1	0	0
C	0	0	1

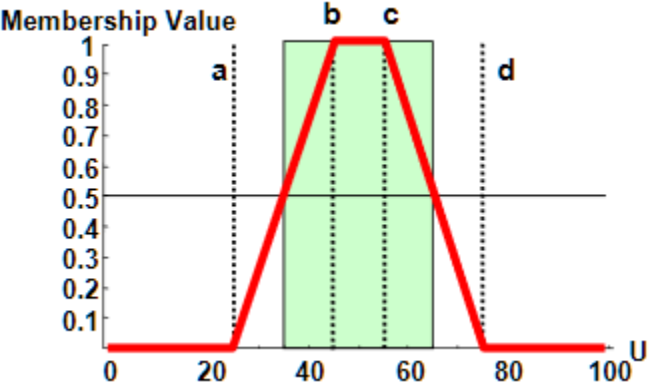
fuzzy

	Short	Average	Tall
A	0.00	0.60	0.50
B	0.50	0.60	0.00
C	0.00	0.56	0.53

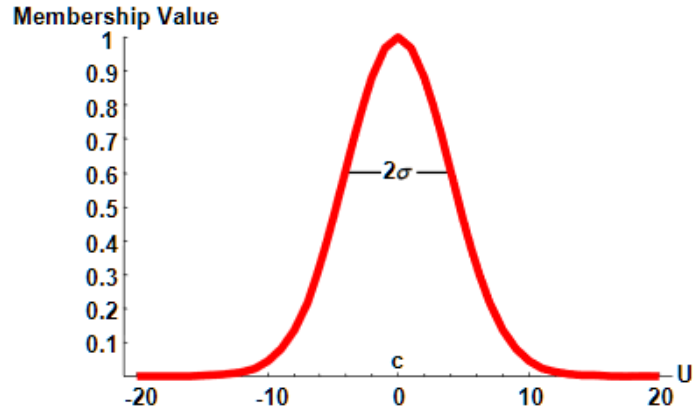
Fuzzy membership functions

Transform “crisp” values into values between 0 and 1, indicating strength of membership in a set...

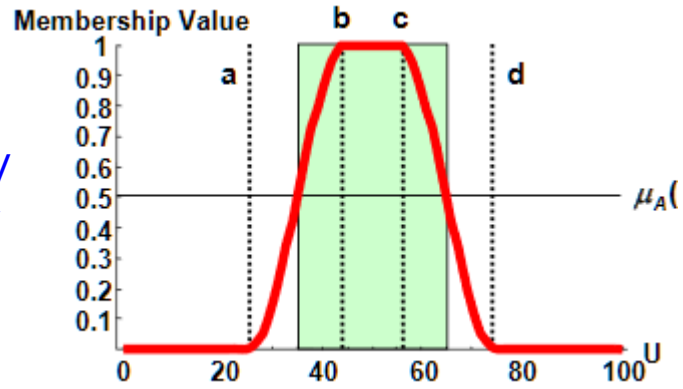
Linear



Gaussian



Sinusoidal/
Sigmoidal/
'S'-shaped



Fuzzy inference

- In binary logic:
 - Premise 1: *If $x = A$, then $y = B$*
 - Premise 2: x is A
 - Conclusion: y is B

- If grass is wet, it has rained.
- The grass is wet, therefore it has rained.



Fuzzy inference

- In fuzzy logic:
 - Premise 1: *If $x = A$, then $y = B$*
 - Premise 2: *x is A' (an acceptably likely member of A)*
 - Conclusion: *y is B' (an acceptably likely member of B)*
- If grass is long, I should mow.
- The grass is *fairly* long, therefore *maybe* I'll mow
- The grass is *really* long, I'd better mow right now!



Fuzzy inference: more conditions

If grass is long and it isn't too hot, I should mow?

Rules:

- Grass is long, not too hot: I should mow now!
- Grass is not very long, not too hot: I'll mow later.
- Grass is long, too hot: I may mow now.
- Grass is not very long, too hot: I'll mow later.

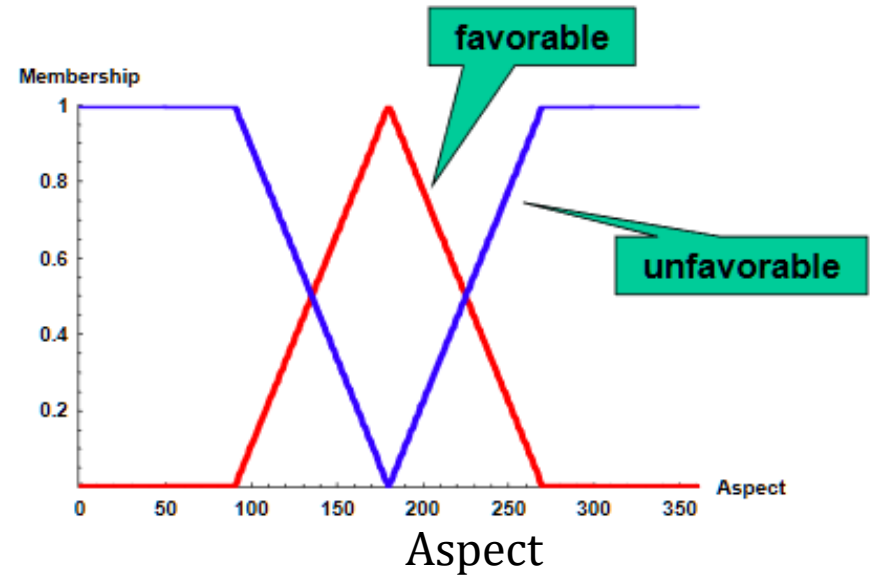
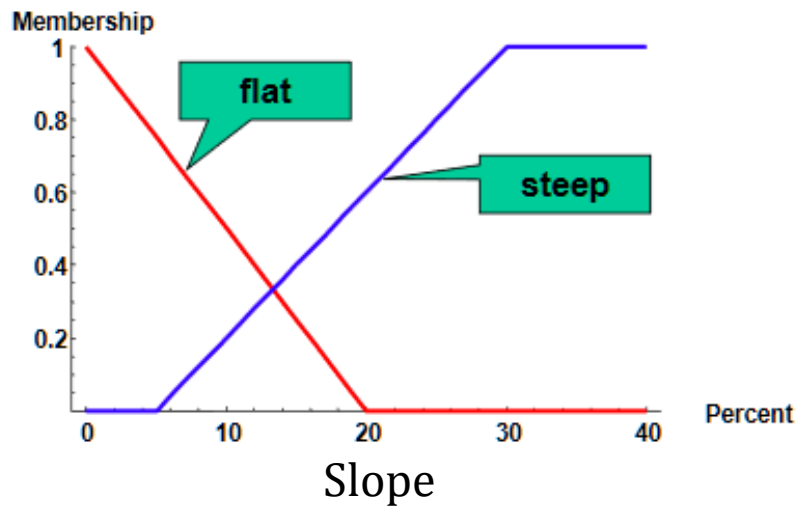
Another example...

Risk analysis based on degrees of risk ranging from 1 (low risk) to 4 (very high risk).

- If **slope is flat** & **aspect is favorable**, then risk is **1**.
- If **slope is steep** & **aspect is favorable**, then risk is **2**.
- If **slope is flat** & **aspect is unfavorable**, then risk is **1**.
- If **slope is steep** & **aspect is unfavorable**, then risk is **4**.

Another example...

Fuzzy memberships:



Slope of 10% and aspect of 180°

	Slope (s)	Aspect (a)
Rule1	0.5	1
Rule2	0.2	1
Rule3	0.5	0
Rule4	0.2	0

Computing overall scores..

- Compute the minimum score (conservative)
- “Conclusion” (or consequence) is risk * score

Rule 1...risk is 1.
Rule 2...risk is 2.
Rule 3...risk is 1.
Rule 4...risk is 4.

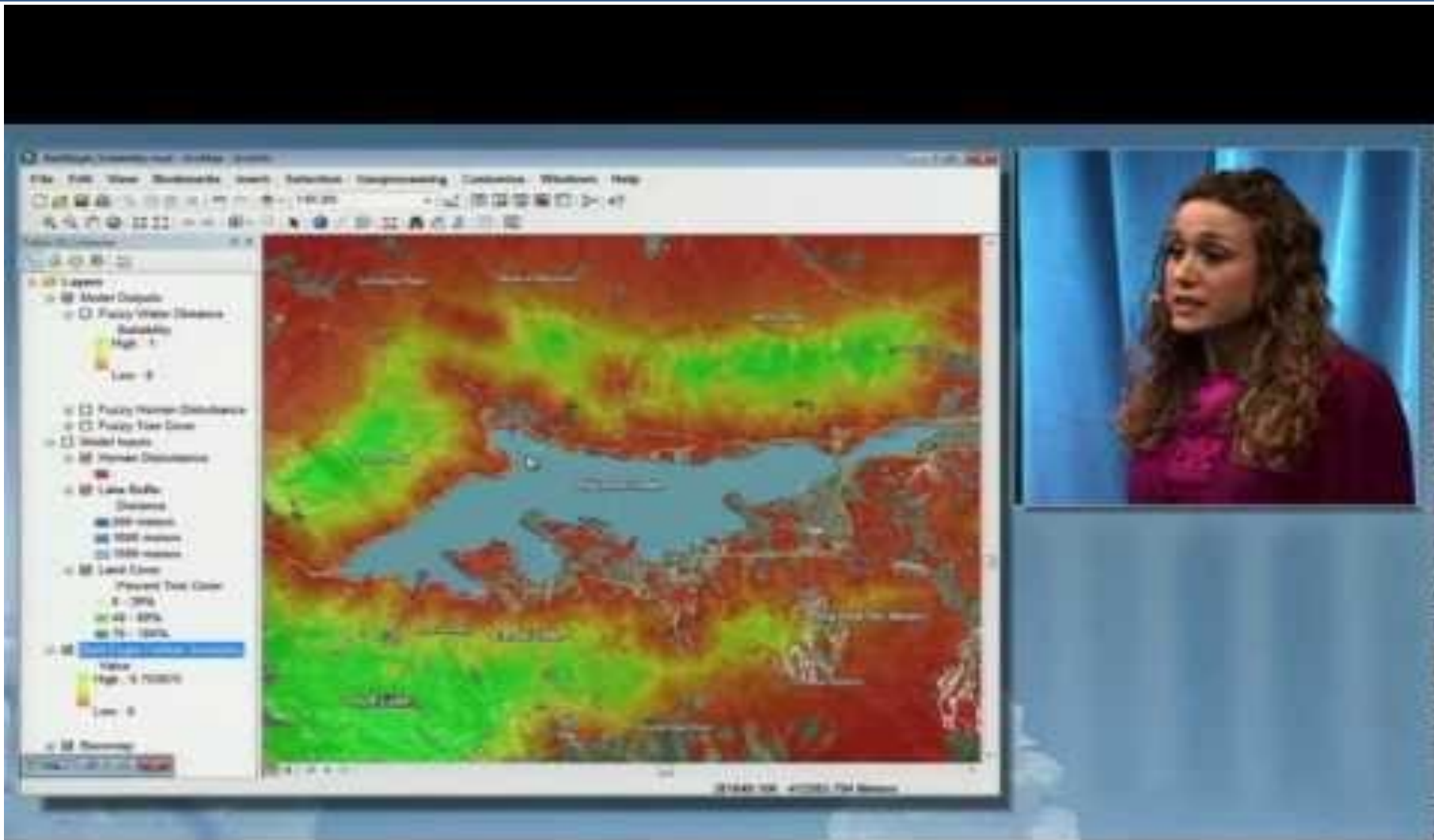
For a slope of 10 percent and an aspect of 180 degrees we have the following results:

	Slope (s)	Aspect (a)	Min(s,a)	Conclusions
Rule1	0.5	1	0.5	0.5
Rule2	0.2	1	0.2	0.4
Rule3	0.5	0	0	0
Rule4	0.2	0	0	0

For the final result we get $c' = \frac{0.5+0.4+0+0}{0.5+0.2+0+0} = 1.29$, which means a low risk.

- *Final result*: divide $\Sigma(\text{conclusion})$ by $\Sigma(\text{min})$
 - It's a weighted sum of the risk (1 thru 4 in this case)

Fuzzy Analysis in ArcGIS



<https://www.youtube.com/watch?v=Hd13H0X00LU>

Bald eagle habitat criteria

- *Land cover*
 - “Not *too sparse* and not *too dense*...”
 - Areas within 40 and 70% forest cover...
- *Water*
 - “Like to be *close* to water”
 - 500, 1000, 1500 m buffer...
- *Human disturbance*
 - “*Far from* from urban areas, roads, etc...”

Bald eagle habitat criteria

Binary logic:

Resulting pixel values are either 1 or 0

Habitat = 1 if:

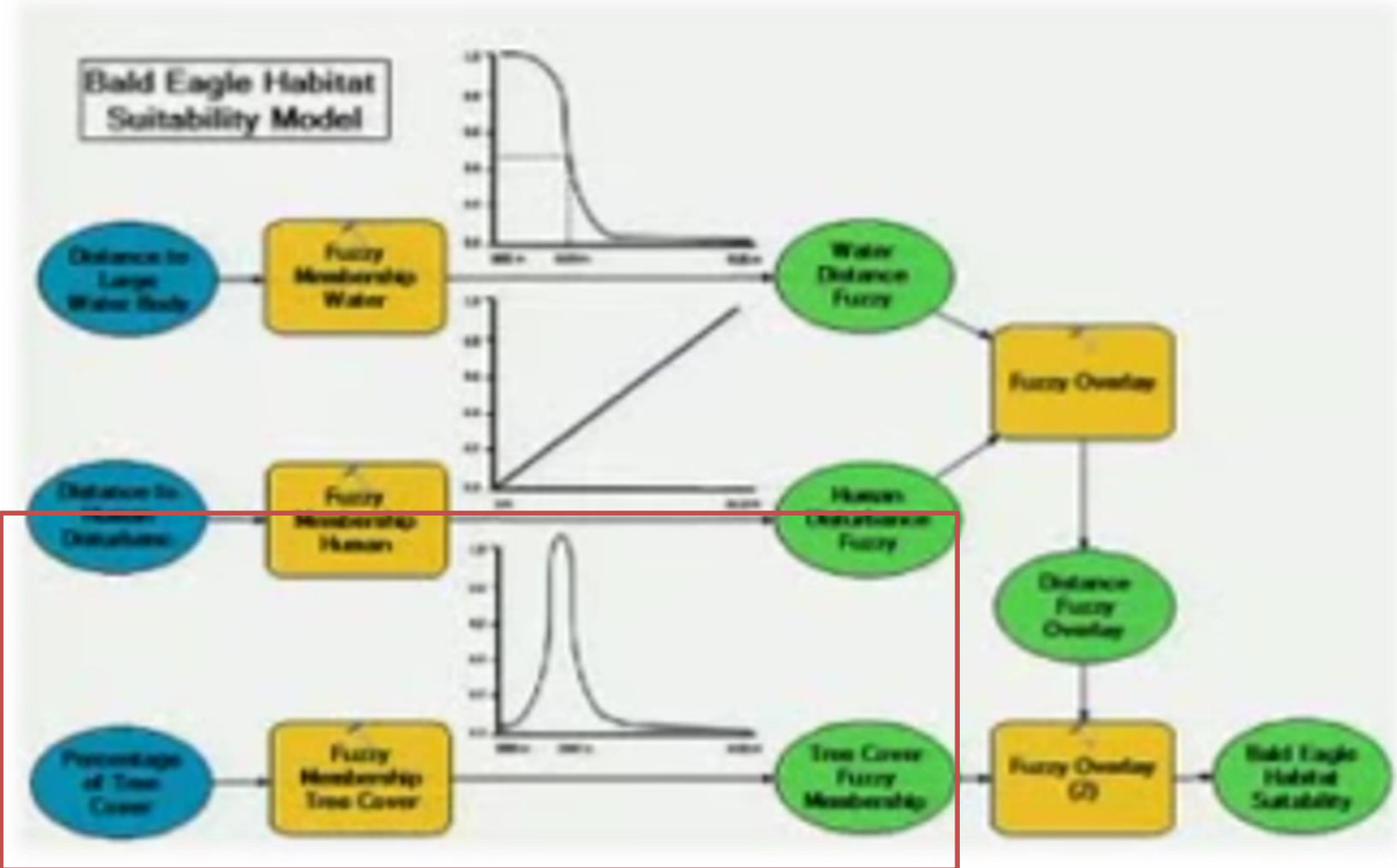
- > 40% forest
&
- < 70% forest
&
- < 1000 m of lake or river
&
- > 1500 m from developed area

Otherwise, Habitat = 0

Bald eagle habitat criteria

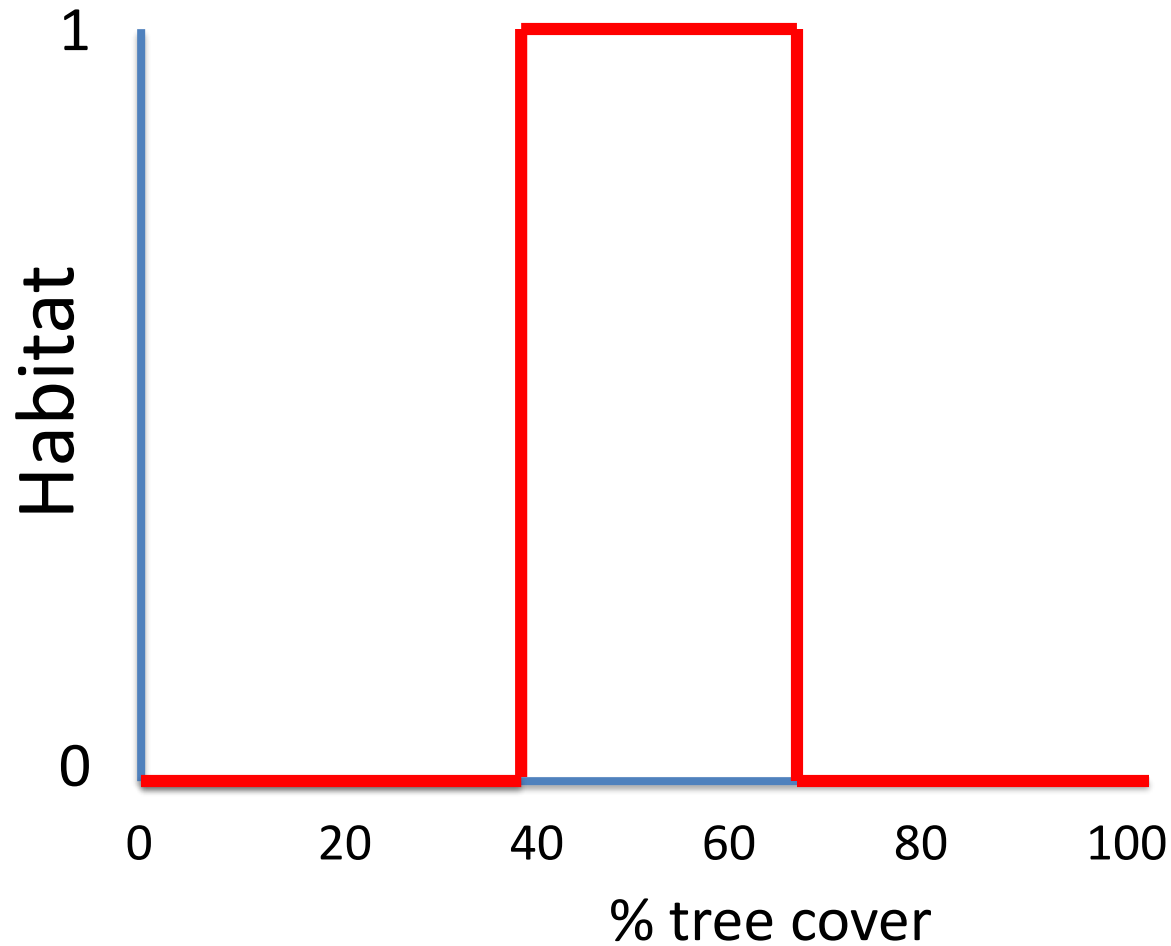
Fuzzy logic:

Results are a continuum between 1 and 0



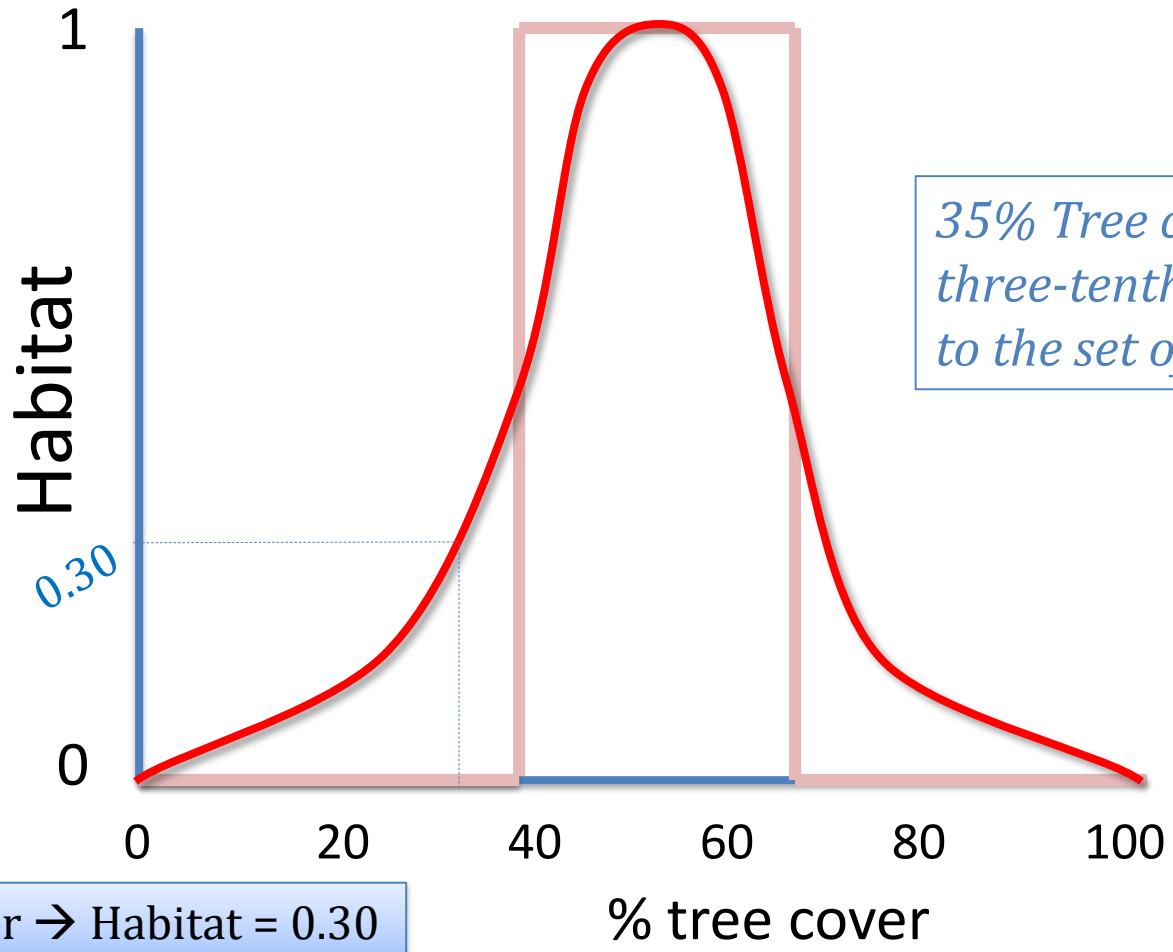
Bald eagle habitat criteria

Binary logic - Forest:



Bald eagle habitat criteria

Fuzzy logic - Forest:



Fuzzy membership functions - ArcMap

- [-] Spatial Analyst Tools
 - [+] Conditional
 - [+] Density
 - [+] Distance
 - [+] Extraction
 - [+] Generalization
 - [+] Groundwater
 - [+] Hydrology
 - [+] Interpolation
 - [+] Local
 - [+] Map Algebra
 - [+] Math
 - [+] Multivariate
 - [+] Neighborhood
 - [-] Overlay
 - Fuzzy Membership**
 - Fuzzy Overlay
 - Weighted Overlay
 - Weighted Sum

Fuzzy Membership

Input raster
Euclidean Distance to Developed

Output raster
V:\Lab5_ThreatMapping_Fuzzy\Scratch\FuzDist

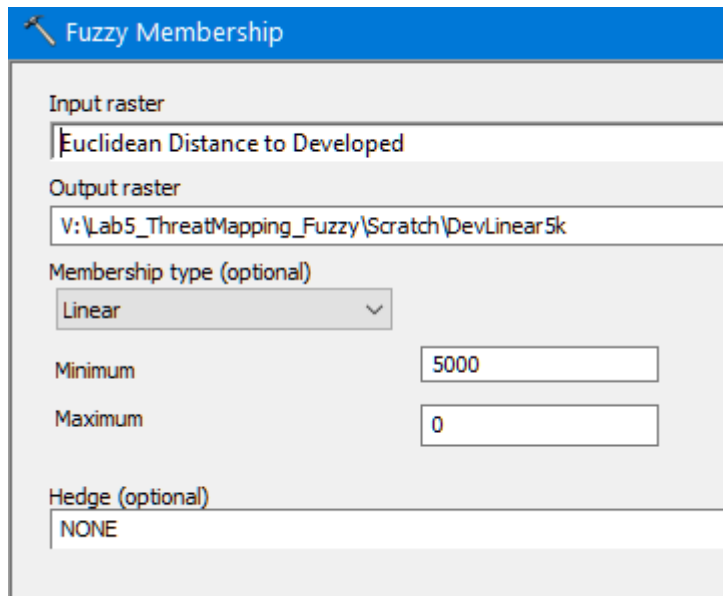
Membership type (optional)

MSSmall	
Gaussian	
Small	1
Large	1
Near	
MSSmall	
MSLarge	
Linear	
NONE	

<http://www.esri.com/news/arcuser/0410/fuzzy-logic-table2.pdf>

Fuzzy membership functions - ArcMap

- Linear:
 - A linear increasing or decreasing membership between two inputs.



The screenshot shows the 'Fuzzy Membership' tool interface. It has a blue header with a hammer icon and the text 'Fuzzy Membership'. Below the header, there are several input fields and a dropdown menu. The 'Input raster' field contains 'Euclidean Distance to Developed'. The 'Output raster' field contains 'V:\Lab5_ThreatMapping_Fuzzy\Scratch\DevLinear5k'. The 'Membership type (optional)' dropdown menu is set to 'Linear'. Below this, there are two input fields: 'Minimum' with the value '5000' and 'Maximum' with the value '0'. At the bottom, the 'Hedge (optional)' field is set to 'NONE'.

Field	Value
Input raster	Euclidean Distance to Developed
Output raster	V:\Lab5_ThreatMapping_Fuzzy\Scratch\DevLinear5k
Membership type (optional)	Linear
Minimum	5000
Maximum	0
Hedge (optional)	NONE

Fuzzy analysis – Pronghorn

- Criteria for unthreatened habitat:
 - Far from developed areas
 - Far from power lines
 - Few nearby roads
 - Mostly open land

Fuzzy analysis – Pronghorn

- Criteria
 - **Far** from developed areas
 - “Far”: > 5000 m, with linear response
 - **Far** from power lines
 - “Far”: > 6000 m, with sigmoidal response
 - **Few** nearby roads
 - “Few”: < 3km per, sq. km with linear response
 - **Mostly** open land cover
 - 60% to 90% herbaceous or scrub within 1km

Demo...

Conclusions

- Fuzzy analysis allows us to soften the impact of somewhat arbitrary cutoffs (e.g. 1km from roads).
- We can do a bit of fuzzy analysis (and did do this) by reclassifying continuous values (e.g. distance from roads) into non-binary classes.
- Alternatively, we can apply fuzzy membership functions, but what function to use involves a different set of assumptions.
- We still need to interpret how to combine the outputs (fuzzy overlay).
- In short, fuzzy analysis doesn't really solve our problem of involving subjective analysis; the subjectivity is just used elsewhere.
- However, fuzzy analysis offers new ways to break down these problems and use decision analysis techniques in our methods.